QUAN 203 - 2021

Tutorial 7: try, scan and upload these exercises to Blackboard before attending your tutorial in Week 11. Those submitted by the notified deadline will be scored and contribute towards the Tutorial Assignment component of your final mark.

1. Evaluate $\sum_{i=1}^{n} e_i$ to show that the residuals in the bivariate regression model sum to zero. What is the interpretation of this result?

2. An estimate of the parameter σ^2 in the standard bivariate regression model is given by
$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n} e_i^2.$$
Noting $e_i = Y_i - \hat{\alpha} - \hat{\beta}X_i$, show that $\sum e_i^2 = S_{YY} - \hat{\beta}S_{XY}$, where $S_{YY} = \sum(Y_i - \bar{Y})^2$ and $S_{XY} = \sum(X_i - \bar{X})(Y_i - \bar{Y})$.

3. Assume the standard bivariate linear regression model, but with $\alpha = 0$ and $\beta = \gamma$, i.e. a line constrained to passing through the origin. Given the residual sum of squares
$$RSS = \sum_{i=1}^{n}(Y_i - \gamma X_i)^2$$
derive the OLS estimate of γ. Do the residuals sum to zero in this case? [You do not need matrix algebra to answer this question.]

4. Let $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ be the "hat" matrix in the multiple regression model, with $\hat{\mathbf{Y}} = \mathbf{HY}$.

   (a) Show that
   - $\mathbf{H}^2 = \mathbf{H}$,
   - $\mathbf{H}(\mathbf{I} - \mathbf{H}) = \mathbf{0}$.
   - $\mathbf{H}' = \mathbf{H}$
   - $(\mathbf{I} - \mathbf{H})'\mathbf{H} = \mathbf{0}$.

   (b) Hence, show that $\mathbf{He} = \mathbf{0}$ where $\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}}$ is the residual vector. If $\mathbf{He}$ is the part of e that can be explained by $\mathbf{X}$, what does this result imply?

   (c) Vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular if $\mathbf{a}'\mathbf{b} = 0$. Confirm that $\mathbf{e}$ and $\hat{\mathbf{Y}}$ are perpendicular.

5. Again, let $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ be the "hat" matrix in the multiple regression model.

   (a) Show that $\mathbf{HX} = \mathbf{X}$. Thus, if $\mathbf{1}$, a column vector of ones, is the first column of $\mathbf{X}$, what is $\mathbf{H1}$?

   (b) Using the results in Q4 and 5a, prove that $\mathbf{e}'\mathbf{1} = 0$ in the multiple regression model if the model includes an intercept.